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LETTER TO THE EDITOR

Magnetotransport of quantum Hall double layers

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Abstract. In a double-well quantum Hall system, electronic motion in either of the two layers introduces a new (pseudo-spin) degree of freedom. The system exhibits a *time-independent* quantized response to the combined effect of two electric fields, one in each layer, of opposite directions. This response is inherently related to itinerant quantum Hall ferromagnetism.

The possibility that a high-mobility electron gas can move in any of the two planes defined by a double quantum well structure (DQWS), under the influence of a strong magnetic field parallel to the well modulation, introduces an additional degree of freedom in the quantum Hall regime [4, 5, 6]. The separation between the layers can be made comparable to the typical separation of electrons within a single layer. This implies that both inter- and intra-layer correlations ought to be considered on a similar footing [3, 2] (for a discussion of the well separated regime see [2]). A growing body of work has dealt with the existence of incompressible liquid states in this system, that are not found in the more conventional single-layer arrangements, the collapse of integer Hall states [4, 3], and the excitations in these systems [3, 7, 2]. References [3] and [2] showed how the layer index can be treated as a pseudo-spin degree of freedom, and how the dominance of the exchange effects of Coulomb repulsion in real space imposes an antisymmetry condition in the spatial part of the wavefunction and therefore implies easy-plane ferromagnetism in the pseudo-spin degrees of freedom.

It recently became possible [10, 1 … o establish *independent* … ic contacts at the two layers, and therefore one can envisage the situation where the s… is subject to external fields that vary from one layer to the other. In this work I will … ut the effect of such a layer-dependent external electric field on the easy-plane ferro… tism of the system in the limit of zero tunnelling.

I will consider a novel proposed [8] magnetotransport experiment that is unique to the DQWS, and show how the system responds as an essentially *itinerant* [2] ferromagnet with a quantized transport coefficient. I study the response of the system to a general electric field $\boldsymbol{E}_\mu$, whose magnitude depends on the layer index $\mu = \uparrow$ or $\downarrow$. In terms of symmetric (antisymmetric) combinations of the fields in each layer $\boldsymbol{E}_\pm = \frac{1}{2}\left(\boldsymbol{E}_\uparrow \pm \boldsymbol{E}_\downarrow\right)$, the linear response to the symmetric field is identical to that of the conventional quantum Hall liquids [8].

In the following, I focus on the interesting case where the electric fields on each layer are opposite to each other $\boldsymbol{E}_+ = \boldsymbol{0}$, $\boldsymbol{E}_- \neq \boldsymbol{0}$, or, equivalently, where chemical potential differences are established at the ends of the two layers (assumed to have a Hall bar geometry) that reverse sign from one layer to the next. Note that it is not in practice necessary for what follows to have the electric fields opposite to each other at each layer.

All that is required is that the two fields be different. The field then can be resolved to a symmetric and an antisymmetric component as above, and the linear response of the system to each component studied. By making the centres of the two wells be at the same potential, the relative chemical potential of the two wells can also be arranged to be on average equal, so that there is no transfer of electric charge from one layer to the other.

I will show that an antisymmetric electric field leads to a non-trivial mixing of charge (spatial) and pseudo-spin degrees of freedom, due to the itinerant nature of the quantum Hall ferromagnet, and a choice of direction in the easy plane of this ferromagnet. In addition, I will demonstrate that, contrary to the prediction of Ho [8], the response is *time independent*, and no space-time vortices are excited. However, the response is quantized in a manner unique to the double layer, that is

$$J_{\uparrow} - J_{\downarrow} = \frac{\nu}{2}\frac{e^2}{h}\hat{B} \times E_{-} \tag{1}$$

where ν is the overall filling factor, i.e. the inverse of the number of flux quanta per electron.

For simplicity, I first consider the case of $\nu = 1$. I will later argue how the results obtained are extended to other filling fractions provided the correlations at those filling factors mirror the correlations built into the so-called (mmm) wavefunctions.

Since I will introduce an electric field that will break rotational invariance in the plane, I work in the Landau gauge $A = (0, -Bx, 0)$ from the beginning (for a magnetic field pointing for convenience along the $-\hat{z}$ axis). The single-particle wavefunctions in the ground Landau level are

$$\psi_{0l}(x, y) = C_k e^{lk_0 lz} e^{-x^2/2} \tag{2}$$

where C_k is a normalization constant ($k = lk_0$), $z = x + \mathrm{i}y$, $k_0 \equiv 2\pi/L_y$ is the minimum separation between the x-coordinates of the guiding centres of the electronic eigenstates (L_y is the size of the system along the y-direction), and l (an integer) is the degeneracy index within each Landau level. Lengths are scaled by the magnetic length $l_B \equiv \sqrt{\hbar c/eB}$. In this gauge, the Laughlin wavefunction for $\nu = 1$ that incorporates intra- and inter-layer correlations on the same footing is

$$\psi_1 = \prod_k \zeta(z_k) \exp\left[-\sum_i x_i^2/2\right] \prod_{i<j} \left(e^{k_0 z_i} - e^{k_0 z_j}\right) \tag{3}$$

where $\zeta(z)$ is the spinor describing the dependence on the layer degree of freedom. Since the Laughlin wavefunction for $\nu = 1$ is a Slater determinant of all the states in the ground Landau level, a gauge transformation accompanied by a change of basis leads from the usual cylindrically symmetric Laughlin wavefunction [1] to the wavefunction above.

The authors of [2] convincingly argued that exchange effects favour a ferromagnetic alignment of the spinors, in the ground state of the double-layer system. A ferromagnetic alignment is also favoured in linear response to an external electric field. Assuming quantum Hall ferromagnetism, I calculate the response of the double layer under an antisymmetric electric field.

Let me briefly consider a finite tunnelling amplitude t that connects single-particle states on each layer with the same guiding centre quantum number k (such states have the largest spatial overlap). I will consider the effect of the electric field in the framework of linear response. An antisymmetric electric field will result in a shift in the energies of the upper and lower layer states with the same quantum number k. Locally then, and rigorously in the limit of vanishing magnetic length, the Hamiltonian that incorporates both tunnelling and the effect of the field separates into block diagonal form, such that for each k it reads

$$H_k = \begin{pmatrix} \epsilon + \delta_k & -t \\ -t & \epsilon - \delta_k \end{pmatrix}$$

where $\epsilon = \hbar\omega_c/2$ is the degenerate energy of the lowest Landau level in the absence of the electric field and δ_k is proportional to the local (at the guiding centre k) potential difference between the two layers, and t is the tunnelling amplitude between the layers. Diagonalizing this Hamiltonian, one sees that the electric field, in linear response, produces a mixing between the symmetric and antisymmetric states, proportional to the energy difference between the upper and lower k states in the antisymmetric electric field.

In the following, I restrict myself to the case $t = 0$, but I will introduce wavefunctions that are appropriate as a zeroth-order approximation to the exact wavefunctions for finite t, in the sense that, for low electric fields (linear response) they mix into the symmetric lowest energy wavefunction, an antisymmetric component with a coefficient proportional to the potential energy difference between states with the same guiding centre x-coordinate.

First consider the one-particle wavefunction:

$$u_k = C_k \begin{pmatrix} \mathrm{e}^{vx} \\ \mathrm{e}^{-vx} \end{pmatrix} \mathrm{e}^{-x^2/2} \mathrm{e}^{k_0 i z} \tag{4}$$

where v is a dimensionless measure of the electric field $v \equiv e|E|l_B/\hbar\omega_c$, and the + (−) sign corresponds to the upper (lower) layer. Each spinor component of this wavefunction is an exact eigenstate of the one-particle problem in the respective layer in the lowest Landau level, including the layer electric field.

I now use these wavefunctions to construct many-body Laughlin states. For filling factor $\nu = 1$, the Laughlin wavefunction coincides with the Slater determinant for a completely filled Landau level. The corresponding wavefunction in the presence of an electric field constructed out of these (equation (4)) single-particle wavefunctions is naturally written in the Landau gauge as a Slater determinant:

$$\psi_{-,1} = \prod_k \begin{pmatrix} \mathrm{e}^{vx_k} \\ \mathrm{e}^{-vx_k} \end{pmatrix} \exp\left[-\sum_l x_l^2/2\right] \prod_{i<j} \left(\mathrm{e}^{k_0 z_i} - \mathrm{e}^{k_0 z_j}\right) \tag{5}$$

where the spinors incorporate the effects of the antisymmetric electric field.

As expected, the uniform antisymmetric field affects only the centre of mass part of the wavefunction, if motion is restricted in a single plane. A nontrivial linear combination of pairs of in-plane quasi-particle excitations is excited by the antisymmetric field, once motion in both planes is allowed. By expanding the spinors to linear order in v, one sees that the effect of the electric field is a mixing of the two orthogonal directions in the easy plane of the ferromagnet, the relative amplitude of which is proportional to the spatial degree of freedom x (and, approximately, to the local potential difference at guiding centre k), and hence the essential *itinerant* nature of the quantum Hall ferromagnet under the antisymmetric field.

The electric current is calculated directly from the wavefunction, as in the standard quantum Hall effect, and in linear response is found to be

$$J_- = \frac{\nu}{2}\frac{e^2}{h}\hat{B} \times E_-. \tag{6}$$

The derivation of this expression is somewhat lengthy, but the physics is very simple. The current in each layer is proportional to the filling factor, as in the standard case, but there is an extra factor of 1/2, which reflects the fact that each electron has a probability of 1/2 of being in each layer. Subtracting the expressions for the current in each layer, yields equation (6).

For filling factors $1/m$ less than unity the Laughlin wavefunction can be written as

$$\psi_{1/m} = \left\{ \exp\left(\sum_k \frac{|z_k|^2}{4m} \right) \prod_{i<j} (z_i - z_j) \right\}^m . \tag{7}$$

Since, this wavefunction is an eigenfunction of angular momentum, and therefore composed of terms each of which is of the same degree in powers of the z_i, this is the wavefunction for $\nu = 1$ raised to the mth power, except that the variables x, y have been scaled by $1/\sqrt{m}$. By performing the change of basis from the angular momentum basis to the guiding centre basis, we also get a Slater determinant of guiding centre wavefunctions in the scaled variables. The scaling factor in the exponents of the factor

$$\prod_{i<j} \left(e^{k_0 z_i/\sqrt{m}} - e^{k_0 z_j/\sqrt{m}} \right)$$

can be incorporated into $k_0' \equiv k_0/\sqrt{m}$ to lead to a new set of single-particle wavefunctions, appropriate for an area scaled by a factor of m, thereby reducing the filling factor (since the number of particles N is kept fixed throughout) to $1/m$. Therefore the wavefunction that incorporates the energy-reducing correlations of the Laughlin wavefunction in the Landau gauge is [9]

$$\psi_{1/m} = \exp\left[-\sum_i x_i^2/2 \right] \prod_{i<j} \left(e^{k_0' z_i} - e^{k_0' z_j} \right)^m . \tag{8}$$

The addition of the antisymmetric field will not, in linear response, destroy these correlations; therefore

$$\psi_{-,1/m} = \prod_k \begin{pmatrix} e^{vx_k} \\ e^{-vx_k} \end{pmatrix} \exp\left[-\sum_l x_l^2/2 \right] \prod_{i<j} \left(e^{k_0 z_i} - e^{k_0 z_j} \right)^m \tag{9}$$

is a variational ground state function for filling factor $1/m$ (m odd), in the presence of an antisymmetric electric field, as long as inter-layer correlations dominate (which has been shown to be the case by [2]). Thus the standard calculation for the electric current leads to the generalization of equation (1) for filling factors of the form $1/(2p+1)$.

In conclusion I have shown that an antisymmetric electric field induces a novel quantized response in a double-layer quantum Hall system that exemplifies the itinerant nature of the antiferromagnetic properties (in the pseudo-spin) of DQWS. It has been argued [2] that beyond a certain separation d^* between the layers, there is a phase transition to a state for which inter-layer correlations cease to be important. It will be interesting to investigate whether the nature of the transition can be illuminated by the response of the system to an antisymmetric layered field.

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